

## Abstract

KOC, BAHATTIN Computational geometric analysis and planning for 3D rapid prototyping processes. (Under the direction of Professor Yuan-Shin Lee)

In this research, computational geometric analysis and planning methods are developed for rapid prototyping (RP) processes. To increase the accuracy of the slicing contours from the STL file, biarc curve fitting of the linear contours has been developed. The mathematics formulation and a *Max-Fit* algorithm have been developed to find the best biarc curve fitting of the STL slicing data points. *Max-Fit* biarc fitting algorithm progresses through all the points on the slicing contour data to find an efficient biarc fitting. The results show that rough cross-sectional contours from the STL models can be smoothed with the developed method of *Max-Fit* biarc. Therefore, less strict requirements on the STL triangulation tolerance can be used when the STL model is generated for rapid prototyping. To increase the efficiency of the rapid prototyping process, non-uniform offsetting and hollowing of solid objects by using biarcs fitting have been developed. The developed non-uniform offsetting method can reduce the area that needs to be built so the total RP build time can be reduced significantly. Using the developed method can also reduce the expensive build material usage. The hollowed objects with constant wall thickness are constructed by offsetting the vertices in their corresponding normal directions. To reduce the staircase effect in the 2D layers of RP processes, a new approach of ruled layer approximation of STL models has been developed. A surface error analysis method has been developed for the proposed ruled layer approximation. Using the surface error assessment, the STL model can be sliced adaptively to reduce the total number of layers without sacrificing the required surface accuracy. To fabricate the ruled layers, the material removal process, specifically 5-axis machining, is integrated into the traditional 2D RP processes. The tool paths are generated for the ruled layers to machine the thick layers fabricated by a traditional RP process. Computer implementation and illustrative examples of the proposed methods are also presented in this dissertation. The results show the developed methods can significantly increase the accuracy and the efficiency of RP processes.

# **Computational Geometric Analysis and Planning for 3D Rapid Prototyping Processes**

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